

Math 72: Selected Review Chapters 5-6

- 1) Find the domain of the rational function. $f(x) = \frac{x^2+2x-3}{2x^2+9x+9}$
- 2) Find the x-intercept(s) of the rational function. $f(x) = \frac{x^2+2x-3}{2x^2+9x+9}$
- 3) Find the y-intercept(s) of the rational function. $f(x) = \frac{x^2+2x-3}{2x^2+9x+9}$
- 4) Simplify the rational expression. $\frac{x^2+2x-3}{2x^2+9x+9}$
- 5) Add the rational expressions. $\frac{x+3}{2x+3} + \frac{x-1}{x+3}$
- 6) Subtract the rational expressions. $\frac{x+3}{2x+3} - \frac{x-1}{x+3}$
- 7) Multiply the rational expressions. $\frac{x+3}{2x+3} \cdot \frac{x-1}{x+3}$
- 8) Divide the rational expressions. $\frac{x+3}{2x+3} \div \frac{x-1}{x+3}$
- 9) Divide. $\frac{x^2+2x-3}{x+9}$
- 10) Perform the indicated operations and simplify. $\frac{2x+3}{x+3} + \left(\frac{x-1}{x+3}\right)^2 \div \frac{x-1}{x+3}$
- 11) Simplify the complex fraction. $\frac{\frac{x-1}{x+3}}{\frac{x+3}{2x+3}}$
- 12) Solve the rational equation. $\frac{x^2+2x-3}{2x^2+9x+9} = 0$
- 13) Solve the rational equation and round to the nearest hundredth. $\frac{x^2+2x-3}{2x^2+9x+9} = 1$
- 14) Shelly can cut the lawn with a riding mower in 3 hours less time than it takes William to cut the lawn with a push mower. If they can cut the lawn in 4 hours working together, find how long it takes for each to cut the lawn alone. Write an algebraic equation and use your graphing calculator to solve. Round to the nearest hundredth of an hour.

Math 72 Solutions to Group Work chapters 5-6 Selected Review

① Find the domain of $f(x) = \frac{x^2 + 2x - 3}{2x^2 + 9x + 9}$

Domain is all real numbers except those that make the denominator zero, causing divide by zero.

Denom $\neq 0$

$$2x^2 + 9x + 9 = 0$$

$$2x^2 + 6x + 3x + 9 = 0$$

$$2x(x+3) + 3(x+3) = 0$$

$$(x+3)(2x+3) = 0$$

$$\downarrow \quad \downarrow$$

$$x \neq -3 \quad 2x+3=0$$

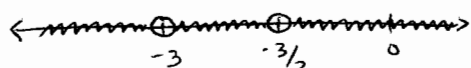
$$2x = -3$$

$$x \neq -\frac{3}{2}$$

$$\begin{array}{r} 2 \cdot 9 \\ 18 \\ 6 \times 3 \\ 9 \end{array}$$

Domain $\{x: x \in \mathbb{R}, x \neq -3, x \neq -\frac{3}{2}\}$

set notation



$(-\infty, -3) \cup (-3, -\frac{3}{2}) \cup (-\frac{3}{2}, \infty)$

interval notation

② Find the x-intercept(s) of $f(x) = \frac{x^2 + 2x - 3}{2x^2 + 9x + 9}$

Points on x-axis have $y=0$.
Replace $f(x)=y=0$.

$$0 = \frac{x^2 + 2x - 3}{2x^2 + 9x + 9}$$

cross-multiply
(or multiply by LCD)

$$0 \cdot (2x^2 + 9x + 9) = x^2 + 2x - 3$$

$$0 = x^2 + 2x - 3$$

$$0 = (x+3)(x-1)$$

$$\downarrow \quad \downarrow$$

$$x = -3 \quad x = 1$$

$$\begin{array}{r} -3 \\ 3 \times -1 \\ 2 \end{array}$$

EXCEPT... $x = -3$ is not in the domain!

x-intercept $(1, 0)$

③ Find the y-intercepts of $f(x) = \frac{x^2+2x-3}{2x^2+9x+9}$

Points on the y -axis have $x=0$.

$$f(0) = \frac{0^2 + 2(0) - 3}{2(0)^2 + 9(0) + 9}$$

$$= \frac{-3}{9}$$

$$= -\frac{1}{3}$$

y-intercept $(0, -\frac{1}{3})$

④ Simplify the rational expression $\frac{x^2+2x-3}{2x^2+9x+9}$

Factor $x^2 + 2x - 3$

$$\begin{array}{c} -3 \\ 3 \quad \times \quad -1 \\ 2 \end{array} \quad (x+3)(x-1)$$

Factor $2x^2 + 9x + 9$

$$\begin{array}{r}
 18 \\
 6 \quad \diagdown \quad \diagup \quad 3 \\
 \quad \quad 9
 \end{array}$$

$$\frac{x^2 + 2x - 3}{2x^2 + 9x + 9} = \frac{\cancel{(x+3)}(x-1)}{\cancel{(x+3)}(2x+3)} = \boxed{\frac{x-1}{2x+3}}$$

⑤ Add $\frac{x+3}{2x+3} + \frac{x-1}{x+3}$

need common denominator

$$= \frac{x+3}{2x+3} \cdot \frac{x+3}{x+3} + \frac{x-1}{x+3} \cdot \frac{2x+3}{2x+3}$$

$$= \frac{(x+3)(x+3) + (x-1)(2x+3)}{(x+3)(2x+3)}$$

FoIL twice
to simplify numerator

$$= \frac{x^2 + 3x + 3x + 9 + 2x^2 + 3x - 2x - 3}{(x+3)(2x+3)}$$

$$= \boxed{\frac{3x^2 + 7x + 6}{(x+3)(2x+3)}}$$

Factor numerator

$$\begin{array}{c} 18 \\ \times 7 \\ \hline \end{array}$$

1, 18 $\rightarrow$ 19
2, 9 $\rightarrow$ 11
3, 6 $\rightarrow$ 9
no other options.

Numerator does not factor.

⑥ Subtract $\frac{x+3}{2x+3} - \frac{x-1}{x+3}$

need common denominator

$$= \frac{x+3}{2x+3} \cdot \frac{x+3}{x+3} - \frac{x-1}{x+3} \cdot \frac{2x+3}{2x+3}$$

↑
mult by 1

$$= \frac{x^2 + 3x + 3x + 9 - (2x^2 + 3x - 2x - 3)}{(x+3)(2x+3)}$$

FoIL twice

* Must use ()
to distribute negative

$$= \frac{x^2 + 6x + 9 - (2x^2 + x - 3)}{(x+3)(2x+3)}$$

combine

$$= \frac{x^2 + 6x + 9 - 2x^2 - x + 3}{(x+3)(2x+3)}$$

dist neg

$$= \frac{-x^2 + 5x + 12}{(x+3)(2x+3)}$$

Factor numerator

$$-(x^2 - 5x - 12)$$

$$\begin{array}{c} -12 \\ \times -5 \\ \hline \end{array}$$

-12, +1 $\Rightarrow$ -11
-6, +2 $\Rightarrow$ -4
-4, +3 $\Rightarrow$ -1
no other options

$$= \boxed{\frac{-(x^2 - 5x - 12)}{(x+3)(2x+3)}}$$

⑦ Multiply $\frac{x+3}{2x+3} \cdot \frac{x-1}{x+3}$

$$= \frac{(\cancel{x+3})(x-1)}{(2x+3)(\cancel{x+3})}$$

$$= \boxed{\frac{x-1}{2x+3}}$$

⑧ Divide $\frac{x+3}{2x+3} \div \frac{x-1}{x+3}$

$$= \frac{(x+3)}{(2x+3)} \cdot \frac{(x+3)}{(x-1)}$$

$$= \boxed{\frac{(x+3)^2}{(2x+3)(x-1)}}$$

nothing cancels!

⑨ Divide $\frac{x^2+2x-3}{x+9}$

$$\begin{array}{r|rrr} -9 & 1 & 2 & -3 \\ & & -9 & 63 \\ \hline & 1 & -7 & 60 \end{array}$$

synthetic division

$$\boxed{x-7+\frac{60}{x+9}}$$

⑩ $\frac{2x+3}{x+3} + \left(\frac{x-1}{x+3}\right)^2 \div \frac{x-1}{x+3}$

$$= \frac{2x+3}{x+3} + \frac{(x-1)(x-1)}{(x+3)(x+3)} \div \frac{x-1}{x+3}$$

$$= \frac{2x+3}{x+3} + \frac{(x-1)(\cancel{x-1})}{(x+3)(\cancel{x+3})} \cdot \frac{(\cancel{x+3})}{(\cancel{x-1})}$$

order of operations
exponent before divide

divide before add

$$= \frac{2x+3}{x+3} + \frac{x-1}{x+3}$$

$$= \frac{2x+3+x-1}{(x+3)}$$

$$= \boxed{\frac{3x+2}{x+3}}$$

already have a common denominator

$$\textcircled{11} \quad \frac{\frac{x-1}{x+3}}{\frac{x+3}{2x+3}}$$

Method 1:

$$\frac{x-1}{x+3} \div \frac{x+3}{2x+3}$$

$$= \frac{x-1}{x+3} \cdot \frac{2x+3}{x+3}$$

$$= \boxed{\frac{(x-1)(2x+3)}{(x+3)^2}}$$

Method 2:

$$\frac{\frac{x-1}{\cancel{x+3}} \cdot \frac{\cancel{(x+3)}(2x+3)}{1}}{\frac{x+3}{\cancel{2x+3}} \cdot \frac{(x+3)\cancel{(2x+3)}}{1}}$$

$$= \boxed{\frac{(x-1)(2x+3)}{(x+3)^2}}$$

mult by $\frac{LCD}{LCD} = 1$

$$(12) \frac{x^2 + 2x - 3}{2x^2 + 9x + 9} = 0.$$

mult by LCD or cross-multiply

$$x^2 + 2x - 3 = 0 \cdot (2x^2 + 9x + 9)$$

$$x^2 + 2x - 3 = 0$$

$$\begin{array}{r} -3 \\ 3 \times -1 \\ 2 \end{array}$$

$$(x+3)(x-1) = 0$$

$$x+3=0 \quad x-1=0$$

$$x \neq -3$$

$$\boxed{x=1}$$

reject

extraneous

denom $\neq 0$

$$2x^2 + 9x + 9$$

$$\begin{array}{r} 18 \\ 6 \times 3 \\ 9 \end{array}$$

$$2x^2 + 6x + 3x + 9$$

$$2x(x+3) + 3(x+3)$$

$$(x+3)(2x+3)$$

$$x+3 \neq 0 \quad 2x+3 \neq 0$$

$$x \neq -3 \quad 2x \neq -3$$

$$x \neq -\frac{3}{2}$$

$$(13) \frac{x^2 + 2x - 3}{2x^2 + 9x + 9} = 1$$

mult by LCD or cross-multiply

$$x^2 + 2x - 3 = 1(2x^2 + 9x + 9)$$

$$x^2 + 2x - 3 = 2x^2 + 9x + 9$$

$$\begin{array}{r} -x^2 \quad -2x \quad +3 \\ -x^2 \quad -2x \quad +3 \end{array}$$

$$0 = x^2 + 7x - 6$$

$$\text{GC } \boxed{y=}$$

$$y_1 = x^2 + 7x - 6$$

$$\begin{array}{r} -6 \\ 7 \end{array}$$

denom $\neq 0$

same as (12)

$$x \neq -3$$

$$x \neq -\frac{3}{2}$$

$$-1, 6 \rightarrow 5$$

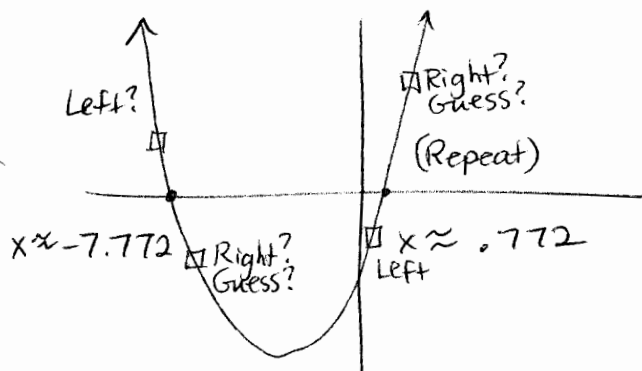
$$-2, 3 \rightarrow 1$$

$$-3, 2 \rightarrow -1$$

$$-6, 1 \rightarrow -5$$

not possible
to factor!

CALC
2nd TRACE
2. Zero



$$\boxed{\begin{array}{l} x \approx -7.77 \\ x \approx 0.77 \end{array}}$$

14 Shelly's time = William's time - 3

$$S = W - 3$$

Work rates

$$\frac{1}{\text{alone}} + \frac{1}{\text{alone}} = \frac{1}{\text{together}}$$

$$\frac{1}{W-3} + \frac{1}{W} = \frac{1}{4}$$

mult all by LCD = $4W(W-3)$

$$\frac{1}{\cancel{W-3}} \cdot \frac{4\cancel{W(W-3)}}{1} + \frac{1}{\cancel{W}} \cdot \frac{4\cancel{W(W-3)}}{1} = \frac{1}{\cancel{4}} \cdot 4W(W-3)$$

$$4W + 4(W-3) = W(W-3)$$

$$4W + 4W - 12 = W^2 - 3W$$

$$8W - 12 = W^2 - 3W$$

$$\underline{-8W + 12} \quad \underline{-8W + 12}$$

$$0 = W^2 - 11W + 12$$

$$\begin{array}{c} 12 \\ - \quad - \\ -11 \end{array}$$

$$-1, -12 \rightarrow -13$$

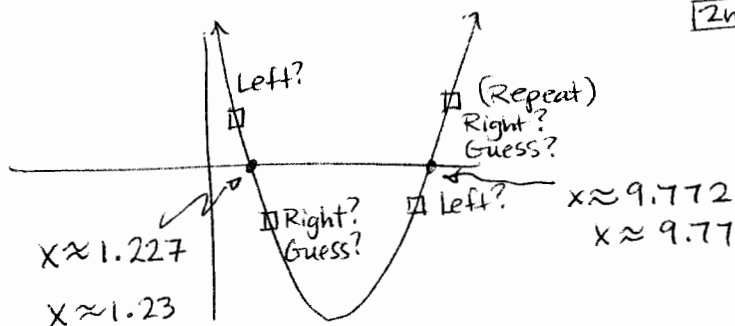
$$-2, -6 \rightarrow -8$$

$$-3, -4 \rightarrow -7$$

not possible.

$Y =$

$$Y_1 = X^2 - 11X + 12$$



CALC
2nd TRACE
2. Zero

If $W = 1.23$

$$\begin{aligned} \text{Then } W-3 &= 1.23-3 \\ &= \text{negative!} \\ &= -1.77 \end{aligned}$$

extraneous

If $W = 9.77$ hrs William

Then $W-3 = 9.77-3 = 6.77$ hrs Shelly